Scalar Potentials and the Swampland

work in progress with Julian Freigang, Dieter Lüst and Guoen Nian

Marco Scalisi

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MAX-PLANCK-INSTITUT
FÜR PHYSIK

Which **scalar potentials** are in the **Swampland**?

Swampland Distance Conjecture

Ooguri, Vafa 2006

"Infinite scalar field variations Δ are always associated to

(at least) an infinite tower of states becoming exponentially light"

$$m \sim m_0 e^{-\lambda\Delta} \qquad \Delta \to \infty$$

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↓

exponential drop-off of the QG cut-off

⚠

$$\Lambda_{QG} = \Lambda_0 e^{-\gamma\Delta}$$

$\Lambda_0 \leq M_P$

original naive cut-off

Swampland Distance Conjecture

upper bound on field displacement

$$\boxed{\Delta < \frac{1}{\lambda} \log \frac{M_P}{\Lambda_{QG}}}$$

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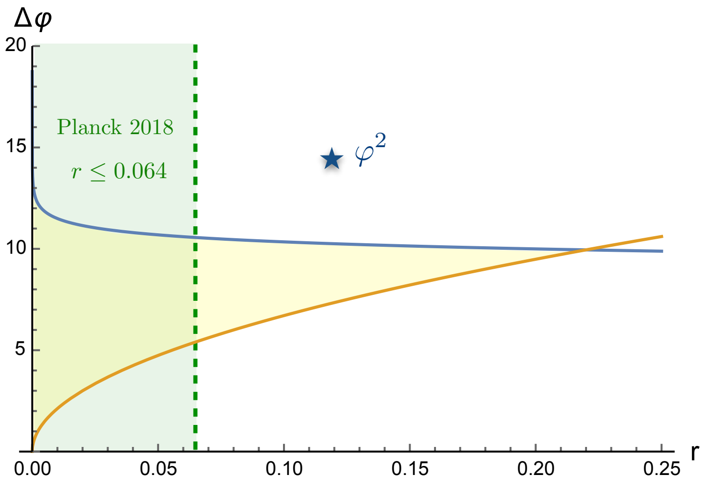
inflation

$$\Delta < \frac{1}{2\lambda}\left(\log \frac{\pi^2 A_s}{2} + \log r\right)$$



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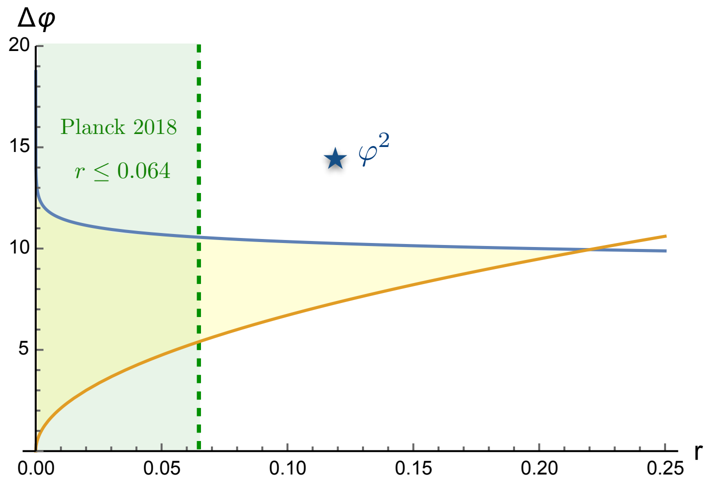

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⚠ **very sensitive** to **decay rate** λ

Decay rate of the SDC

Calderòn-Infante, Uranga, Valenzuela 2020

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$$\lambda(\Delta) = -\frac{d\log m}{d\Delta} = -T^i\partial_i \log m$$

T^i = normalized tangent vector

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$$\lambda = -|\partial \log m| \cos\theta = \lambda_g \cos\theta$$

θ = angle between the trajectory and the geodesic

λ_g = decay rate for geodesics= highest value of λ

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Etheredge et al 2022

see Etheredge's talk

$$\lambda \geq \lambda_0$$

LOWER BOUND

$$\longrightarrow$$

$$\cos\theta \geq -\frac{\lambda_0}{|\partial \log m|} = \frac{\lambda_0}{\lambda_g}$$

MAXIMUM DEVIATION ANGLE

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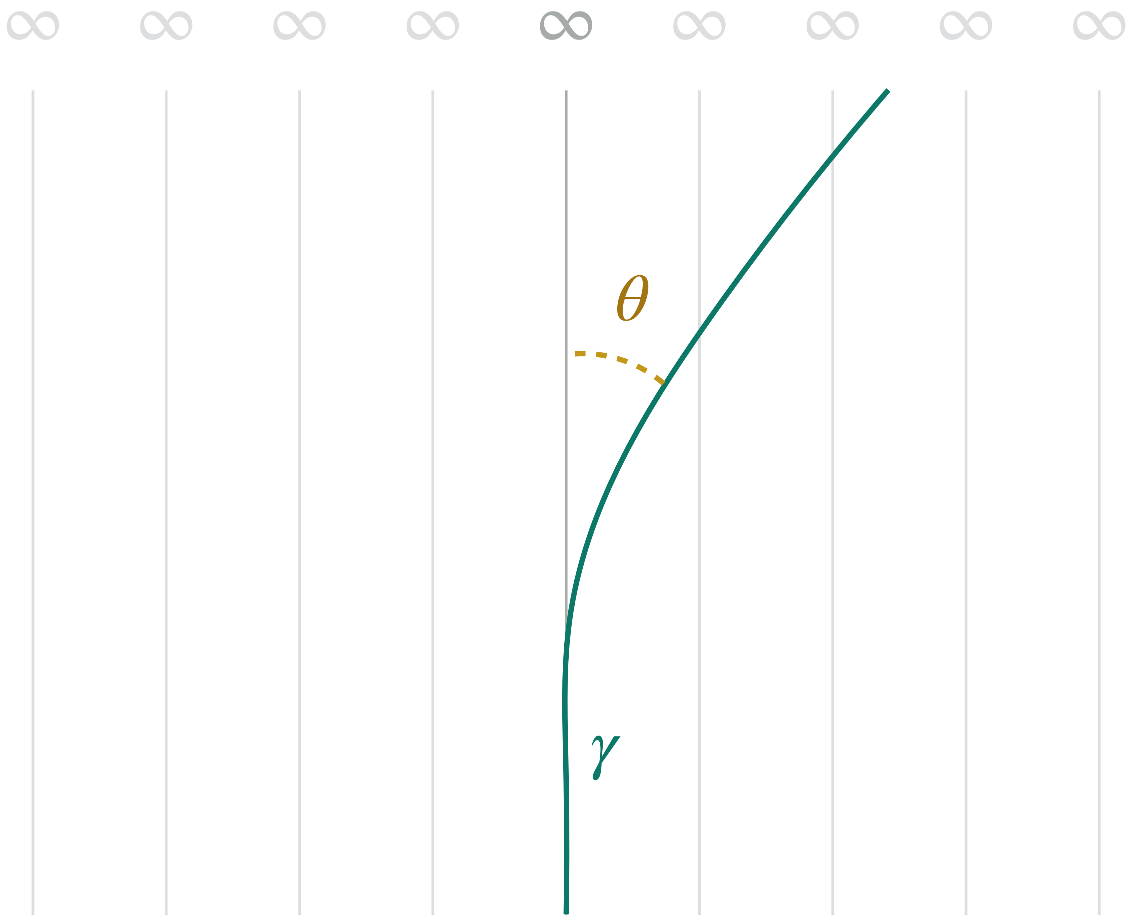

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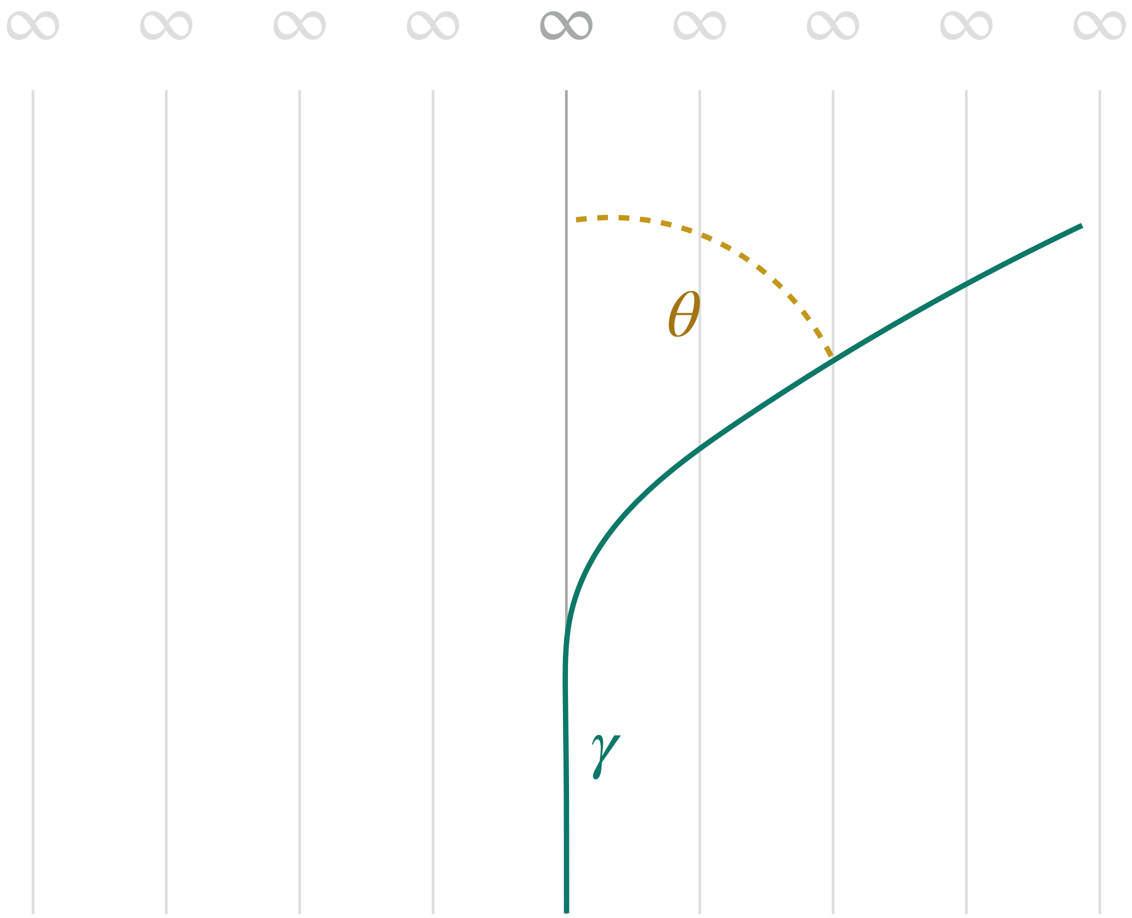

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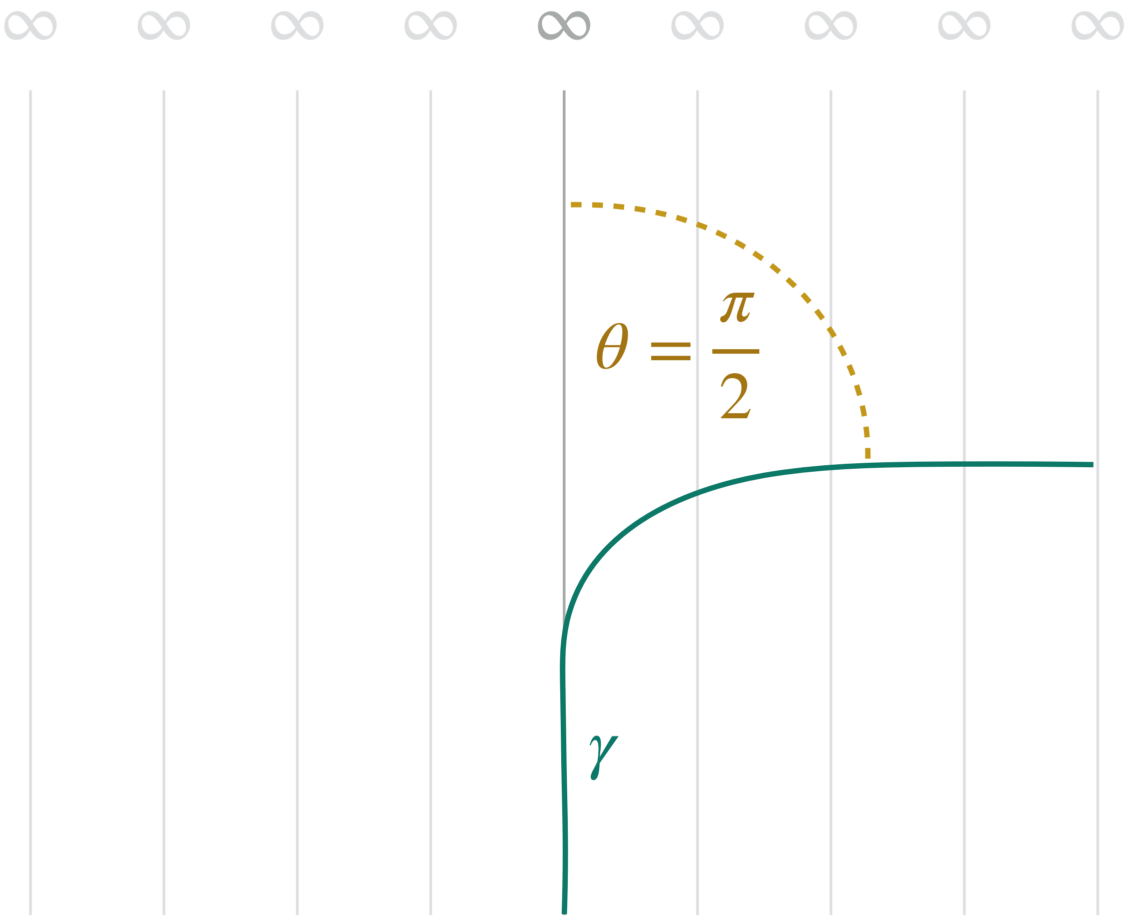

$\lambda = 0$

⚠

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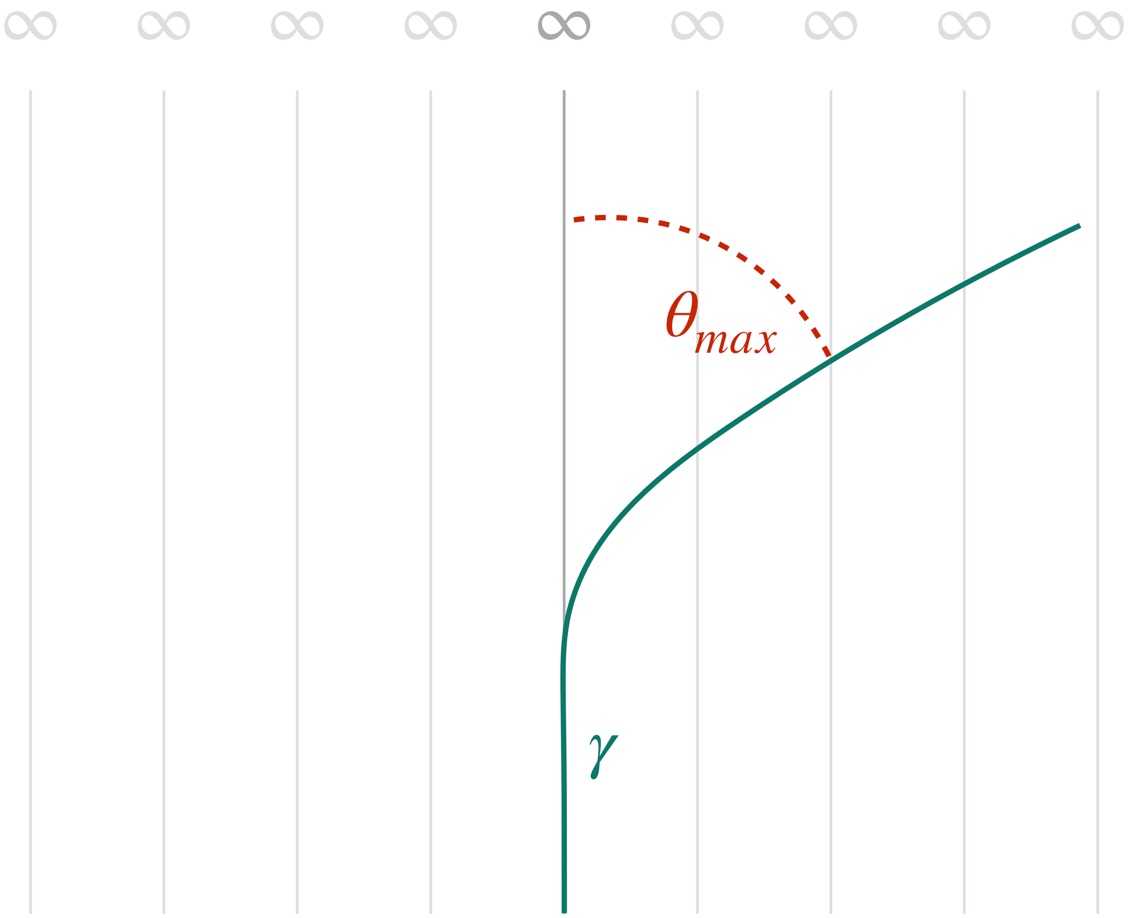

$$\lambda_0 = \lambda_g \cos\theta_{max}$$

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Decay rate in the **axion-saxion model**

*Valenzuela & **MS** 2018*

$$\mathscr{L} = \frac{n^2}{s^2}\left(\dot{s}^2 + \dot{\phi}^2\right)$$

saxion

axion

$$\boxed{(s, \phi) = (s_0 + \delta s, \frac{1}{a}\delta s)}$$

trajectory

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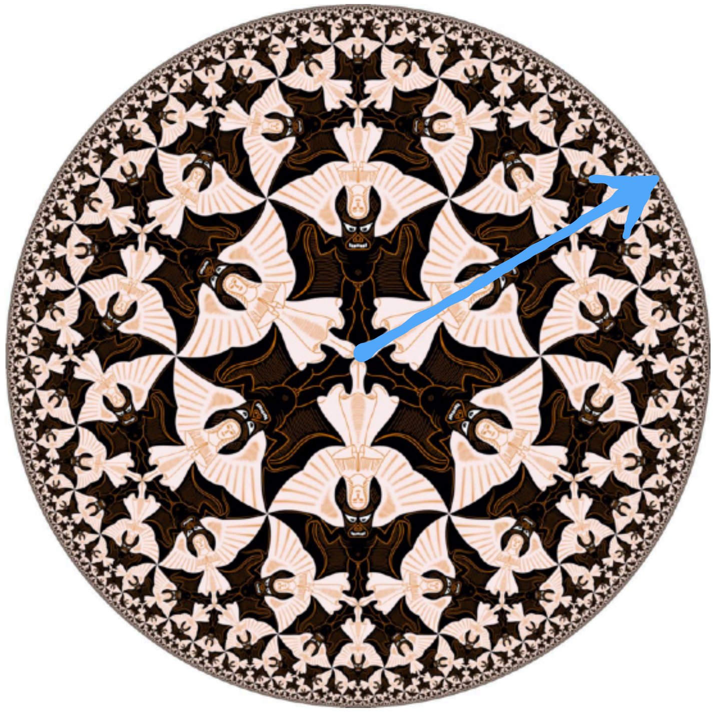

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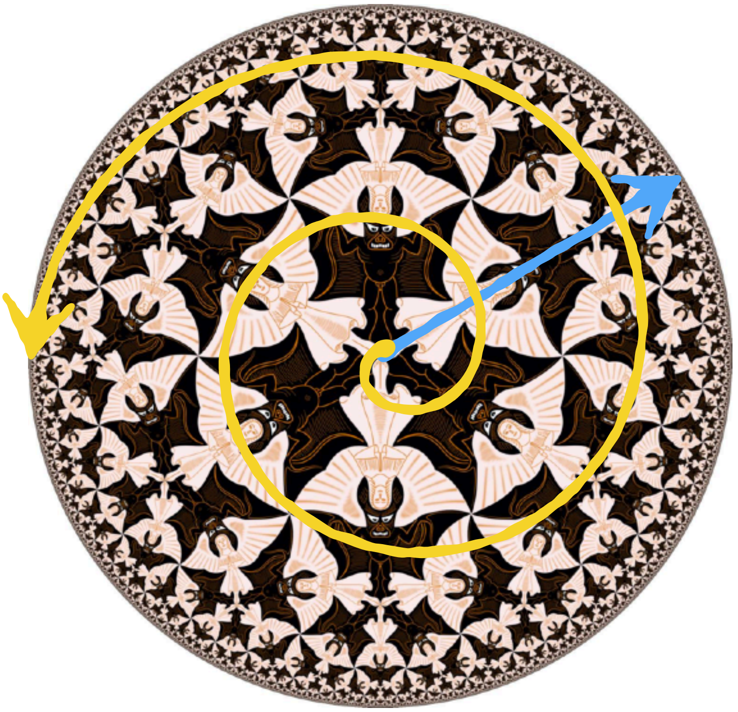

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$a \to \infty$

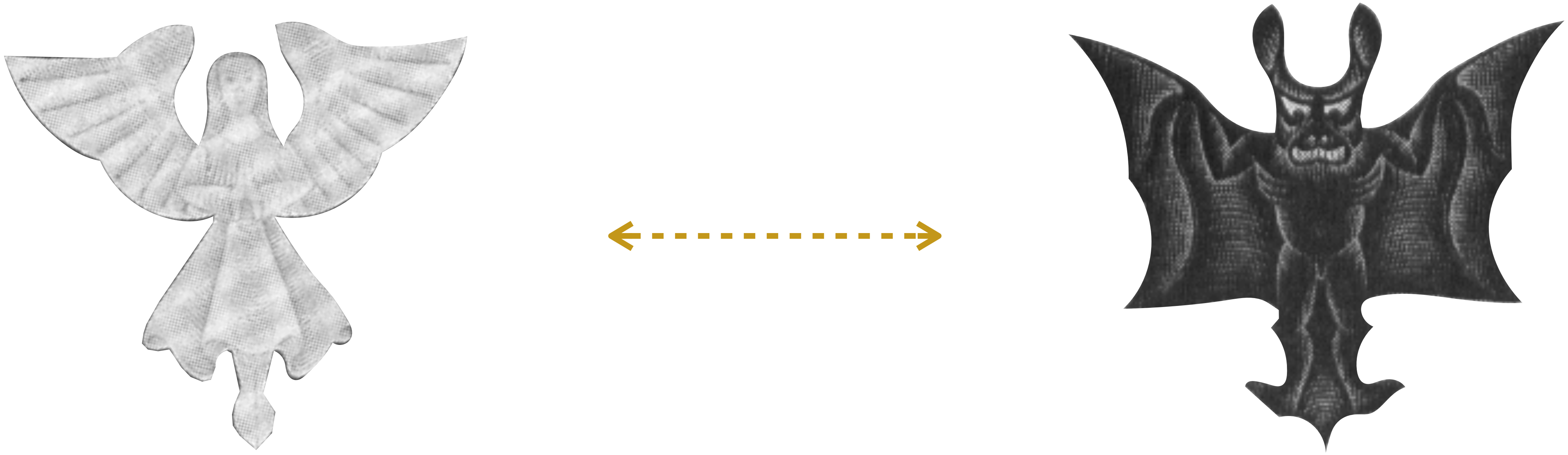
$\lambda_{eff} \sim \lambda$

trajectory mainly saxionic

$a \to 0$

$\lambda_{eff} \ll \lambda$

trajectory mainly axionic



Deviation from geodesics

Scalar potential

Trajectories in moduli space and scalar potentials

- Lagrangian of *n* **free homogeneous scalar fields**

$$\Gamma^i_{jk} = \frac{1}{2} g^{il} \left(\frac{\partial g_{jl}}{\partial \varphi^k} + \frac{\partial g_{kl}}{\partial \varphi^j} - \frac{\partial g_{jk}}{\partial \varphi^l} \right)$$

$$\mathscr{L} = \frac{1}{2} g_{ij}(\varphi) \dot{\varphi}^i \dot{\varphi}^j \dashrightarrow \boxed{\ddot{\varphi}^i + \Gamma^i_{jk} \dot{\varphi}^j \dot{\varphi}^k = 0}$$

EoM
=
geodesic equation

scalar fields will move along geodesics

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- Lagrangian of *n* **scalar fields with a potential**

$$\mathscr{L} = \frac{1}{2} g_{ij}(\varphi) \dot{\varphi}^i \dot{\varphi}^j - V(\varphi) \dashrightarrow \boxed{\ddot{\varphi}^i + \Gamma^i_{jk} \dot{\varphi}^j \dot{\varphi}^k = -g^{il} \frac{\partial V}{\partial \varphi^l}}$$

scalar fields will move under an external conservative force along non-geodesics

Hyperbolic space - setup

$$d\Delta^2 = \frac{n^2}{s^2}\left(ds^2 + d\phi^2\right)$$ **metric**

hyperbolic upper half plane

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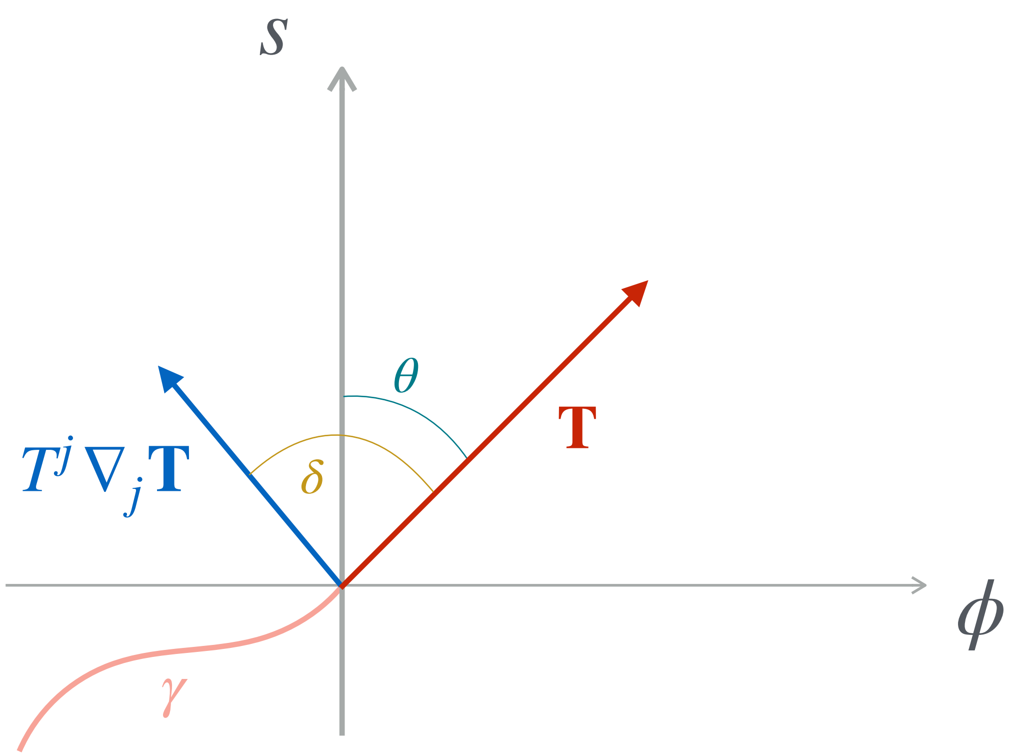


$$\mathbf{T} = \dot{s}\hat{e}_s + \dot{\phi}\hat{e}_\phi$$ **tangent vector**

$$T^j\nabla_j\mathbf{T}$$ **acceleration vector**
(parallel transport of $\mathbf{T}$ along itself)

$$\tan\theta = \frac{\dot{\phi}}{\dot{s}}$$

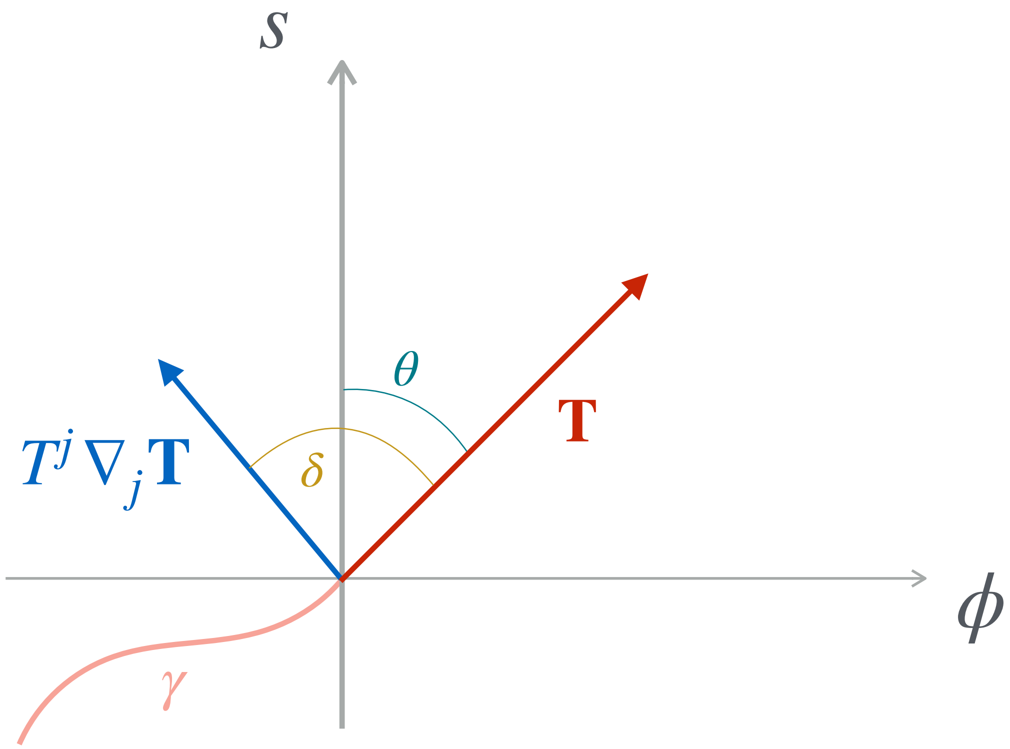
we demonstrate that at infinity $\delta \to \frac{\pi}{2}$

Hyperbolic space - bound on ∇V

$$d\Delta^2 = \frac{n^2}{s^2}\left(ds^2 + d\phi^2\right)$$ **metric**

\+ **scalar potential** $V(s, \phi)$

hyperbolic upper half plane



$$T^j \nabla_j \mathbf{T} = -\nabla V$$ **EoM** (vector form)

$$|\nabla V| = \frac{\sin\theta}{n} = \frac{\sqrt{\lambda_g^2 - \lambda^2}}{n\lambda_g} \leq \frac{\sqrt{\lambda_g^2 - \lambda_0^2}}{n\lambda_g}$$

SDC-constraint on $\mathcal{N} = 1$ supergravity

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- **Hyperbolic Kähler geometry**

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- **Scalar potential** (in terms of superpotential W)

$$V = \frac{1}{(\Phi + \overline{\Phi})^{\alpha}} \left(\frac{(\Phi + \overline{\Phi})^2}{\alpha} \left(\partial W - \frac{\alpha W}{\Phi + \overline{\Phi}} \right) \left(\overline{\partial W} - \frac{\alpha \overline{W}}{\Phi + \overline{\Phi}} \right) - 3\,|\,W\,|^2 \right)$$

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- **Gradient of the potential**

$$|\,\nabla V\,| = \sqrt{\frac{2}{\alpha}}\left(\Phi + \overline{\Phi}\right)^{1-\alpha} |\,W\,|^2\,|\,A\,|$$

$$A \equiv A_1(\Phi + \bar{\Phi})^2 + A_2(\Phi + \bar{\Phi}) + A_3 + A_4(\Phi + \bar{\Phi})^{-1}$$

$$A_1 = \frac{1}{\alpha}\frac{\overline{\partial W}}{\overline{W}}\frac{\partial^2 W}{W} \qquad A_2 = 2\left(\frac{1}{\alpha} - 1\right)\left|\frac{\partial W}{W}\right|^2 - \frac{\partial^2 W}{W}$$

$$A_3 = 2(\alpha - 2)\frac{\partial W}{W} + (\alpha - 1)\frac{\overline{\partial W}}{\overline{W}} \qquad A_4 = \alpha(3 - \alpha)$$

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✗

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$$W = W_0$$

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✗

$$W(\Phi) = a\ e^{b\Phi}$$

$b = b_1 + ib_2$

$b_1 - b_2 \tan\theta < 0$ $\quad |\nabla V| \to 0$

$b_1 - b_2 \tan\theta > 0$ $\quad |\nabla V| \to \infty$

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$$W(\Phi) = c \ln(\Phi)$$

$$|\nabla V| \to 0$$

$$W(\Phi) = l\Phi^m$$

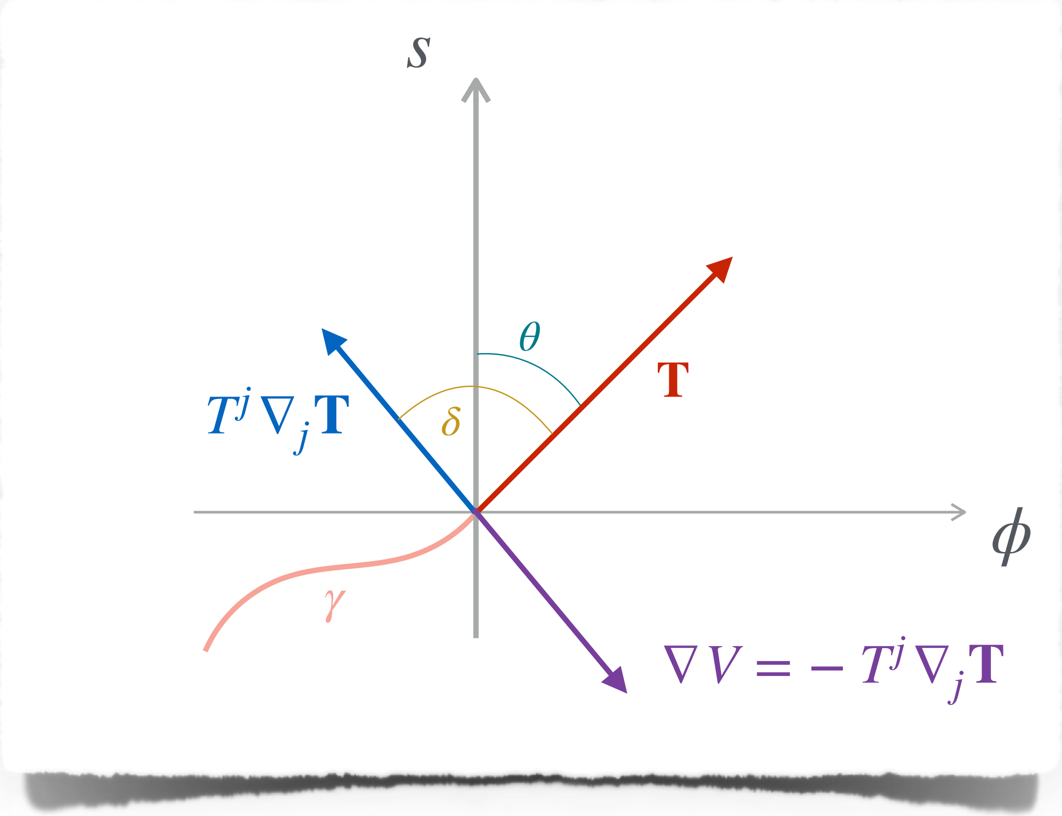
$$|\nabla V| = \sqrt{\frac{2}{\alpha}}\,|l|^2 (2\cos\theta)^{-2m}\left(2s_0 e^{kt}\right)^{2m-\alpha}|\tilde{A}|$$

$2m - \alpha < 0$	$\lvert\nabla V\rvert \to 0$
$2m - \alpha > 0$	$\lvert\nabla V\rvert \to \infty$
$2m - \alpha = 0$	$\lvert\nabla V\rvert \to const$

✓

compatible with **SDC**
and **deviation from geodesic**

SDC-constraint on $\mathcal{N} = 1$ supergravity



$$\partial_s V < 0$$

$$\partial_s V = \frac{|l|^2}{2^\alpha s_0 e^{kt}} (\cos\theta)^{4-\alpha} \tan^2\theta \left(1 - (\alpha - 3)\tan^2\theta\right) < 0$$

$$\boxed{\alpha > 3}$$

$K = -\alpha \ln\left(\Phi + \overline{\Phi}\right)$

bound on Kähler curvature ⚠

Conclusions

We have shown that

the **decay rate $\lambda(\Delta)$** of the SDC

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(see upcoming paper for n-dim)

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we have shown that the only setups **consistent with** the **SDC** and **deviation from geodesics** are

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